THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH2040A (First Term, 2022-23)
Linear Algebra II
Course Outline

Outline

This course is a continuation of Linear Algebra I (MATH1030). It is a second course on linear algebra and will cover basic concepts of abstract vector spaces, linear transformations, eigenvalues and eigenvectors, diagonalizability, operators on inner product spaces, orthogonality and Gram-Schmidt process, adjoint, normal and self-adjoint operators, spectral theorems, and if time permits, quadratic forms and Jordan canonical forms. *More emphasis will be put on the theoretical understanding of basic concepts in linear algebra.*

Prerequisites

Students taking this course should have taken MATH1030 *Linear Algebra I* and MATH1050 *Foundation of Modern Mathematics* (or classes at equivalent level). In particular, students are expected to have good working knowledge on the following:

- solving systems of linear equations by Gaussian eliminations
- matrix manipulations including e.g. rank, inverse, transpose and determinant
- $\mathbb{R}^n$ as a vector space, subspaces, linear span, linear (in)dependence, basis and dimension (computational aspects)
- computing eigenvalues and eigenvectors of a matrix, diagonalization
- inner product on $\mathbb{R}^n$, orthogonality, Gram-Schmidt orthogonalization process
- familiarity with complex numbers and polynomials

Class Information

- Instructor: Prof. Renjun DUAN (Office: LSB 206, Tel: 3943 7977, Email: `rjduan@math.cuhk.edu.hk`; please make an appointment by email if you have any question)
- Lectures: Tuesday 10:30-12:15 Yasumoto Int'l Acad Park LT2; Thursday 16:30-17:15 Lady Shaw Bldg LT3
- Tutorials: Thursday 17:30-18:15 Lady Shaw Bldg LT3
- Webpage: `https://www.math.cuhk.edu.hk/course/2223/math2040a`

Teaching Assistants (Please make an appointment with TAs by email if you have any question)

- Mr. Zongguang LI (Presenter; Office: LSB 232, Tel: 3943 5294, Email: `zgli@math.cuhk.edu.hk`)
- Mr. Kam Fai CHAN (Grader; Office: LSB 232, Tel: 3943 5294, Email: `akfchan@math.cuhk.edu.hk`)

Textbook

- Friedberg, Insel and Spence, *Linear algebra*, 4th edition, Pearson.

Reference

- Axler, *Linear algebra done right*, 3rd edition, Springer.

Lectures, Tutorials and Homeworks

Lectures: Students are supposed to attend ALL the lectures. The lectures will focus mainly on the theoretical concepts and proofs, supplemented occasionally with some illustrative examples. Below you can find a tentative schedule of this course, indicating the involved sections in the textbook that would be covered. As the lectures would only cover the most essential materials (at a rather fast pace), it would be very helpful if you have read (or at least skimmed through) the relevant sections beforehand. The full review note of the course (PDF) would be helpful for you to sketch the covered materials, and students are also strongly encouraged to take your own class notes.

Tutorials: Students are expected to attend ALL the tutorials. The tutorials will cover more examples and computational aspects of the materials. There will be times during the tutorials for discussions and working out some exercises together. All the materials (except otherwise stated) covered in lectures and tutorials will be covered in the midterms and final exam.

Homeworks: There will be WEEKLY problem assignments, usually posted on the course web on Mondays and due on Fridays in the following week. Each problem assignment consists of two parts: the compulsory part and the optional part. You only need to hand in your solutions of the compulsory part. But you are highly recommended to work out the optional part at home as well. Keep in mind that the best way to learn mathematics is to work on exercises and get the feeling by yourself. Apart from the assigned problems, the textbook has a vast collection of exercises at the back of each section. Students are highly encouraged to work out these exercises and turn to TAs if you meet any trouble. Discussions among classmates are strongly encouraged.

Assessment

- 10%: Homework (about ten times)
- 20%: Midterm 1 (**Time and date: 1830-2000 Oct 12 Wed; Venue: LSB LT6**)
- 20%: Midterm 2 (**Time and date: 1830-2000 Nov 16 Wed; Venue: LSB LT6**)
- 50%: Final (TBA by University)

Tentative Schedule (13 weeks in total)

- Week 1–3: Foundations of abstract vector spaces (Textbook Sec. 1.2–1.6)
  - definition of vector spaces and subspaces
  - sum and direct sum of subspaces
  - span and linear independence
  - basis and dimension
- Week 4–6: Linear maps (Textbook Sec. 2.1–2.5)
  - definition of linear maps, linear maps as a vector space
  - null space and range, rank-nullity theorem
  - matrix representation of a linear map, change of basis formula
  - invertibility and isomorphism
  - determinant and its properties (review)
- Week 7–9: Eigenvalues, eigenvectors and diagonalizability (Textbook Sec. 5.1, 5.2, 5.4)
  - eigenvalues and eigenvectors, characteristic polynomials
  - diagonalizability, algebraic and geometric multiplicity
  - invariant subspaces, Cayley-Hamilton theorem
- Week 10–13: Operators on inner product spaces (Textbook Sec. 6.1–6.4, 6.6)
  - inner products and norms, basic identity and inequalities
  - orthogonality, orthonormal basis, orthogonal complement
  - adjoint of an operator
  - normal and self adjoint operators
  - spectral theorems